SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If L is a finite extension of K, K is a finite extension of F, show that L is a finite extension of F.

17. If F is of characteristic zero and if a, b are algebraic over F, show that there is an element $C \in F(a,b)$ such that $F(a,b) = F(c)$.

18. If K is a finite extension of F, show that $G(K,F)$ is a finite group and $O(G(K,F)) \leq [K : F]$.

19. If F is a finite field and $\alpha, \beta \neq 0$ are two elements of F, show that we can find elements a, b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

20. State and prove FROBENIUS theorem.

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APRIL/MAY 2024

DMA21/GMA21 — ALGEBRA II

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define degree of the extension field E over F.

2. Define algebraic element over F.

3. Define splitting field.

4. If $\sigma : F \to F'$ is an isomorphism, show that there is an isomorphism $\sigma^* : F[x] \to F'[t]$.

5. Define fixed field of group of automorphism of K.

6. Define normal extension of F of K.

7. Define solvable group.

8. Let F be a finite field. Show that F has p^m elements, where the prime number p is the characteristic of F.

9. Prove that for all $x, y \in Q$, $N(xy) = N(x)N(y)$.

10. Show that the adjoint in Q satisfies $x^{**} = x$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) If $a, b \in K$ are algebraic over F, show that $a \pm b$, $a\ b$ and $\frac{a}{b}, b \neq 0$ are all algebraic over F.

Or

(b) If $p(x) \in F[x]$ and if K is an extension of F, show that for any element $b \in K$, $p(x) = (x-b)\,q(x) + p(b)$, where $q(x) \in K[x]$ and deg $q(x)$ = deg $p(x)-1$.

12. (a) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non-trivial common factor.

Or

(b) Prove that any two splitting fields of a given polynomial are isomorphic.

13. (a) Show that the fixed field of G is a subfield of K.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomials over F.

14. (a) If $p(x) \in F[x]$ is solvable by radicals over F, prove that Galois group over F of $p(x)$ is a solvable group.

Or

(b) Show that any two finite fields having the same number of elements are isomorphic.

15. (a) If $a \in H$, show that $C^{-1} \in H$ if and only if $N(a) = 1$.

Or

(b) Let C be a field of complex numbers and that the division ring D is algebraic over C, prove that $D = C$.